# MATHEMATICAL MODELING OF ALCOHOLISM INCORPORATING MEDIA AWARENESS

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# DECLARATION

This thesis is my original work and has not been presented for a de	gree in
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#### ABSTRACT

Alcohol addiction is a phenomenon that has attracted the attention of numerous researchers and academics in a variety of professions due to its serious repercussion on all spheres of human life. Alcoholism is a common addiction in adults throughout the globe. To shed light on the causes of this phenomenon and pinpoint efficient preventative and therapeutic measures, a mathematical model that incorporates media awareness and the impact of the exposed class on light and heavy drinkers' is crucial. The impact of media awareness and treatment on the drinking behavior of various drinker classes is discussed in several mathematical models. Nevertheless, the impact of the exposed class of alcoholics on light and heavy drinkers in the presence of media awareness, has not been addressed. There is correlation between the exposed class and awareness over time, hence it is vital to understand the spread of alcoholism and the need to curb it. The main objective of this study is to formulate and analyze, a mathematical model of alcoholism incorporating media awareness and the influence of the exposed class on light and heavy drinkers. A set of differential equations served as the foundation for the model's formulation. To perform stability analysis of the model at each equilibrium point, Jacobian matrix method and the next generation matrix approach were employed. This further aided in the computation of the basic reproduction number,  $R_0$ . By use of MAT-LAB, numerical simulations on the impact of awareness on alcoholism were performed. Secondary data obtained from NACADA and data from rehabilitation centers in Kenya was used to validate the analytical results of the impact of media awareness on the drinking population. Stability analysis of the model indicated that the Alcohol Free Equilibrium (AFE) point is locally asymptotically stable whenever  $R_0 < 1$  and unstable whenever  $R_0 > 1$ . Additionally, the Alcohol Endemic Point (AEP) exists and is locally asymptotically stable when  $R_0 > 1$ . Numerical simulations showed that increase in media awareness programs reduces alcohol prevalence in the community. The study concluded that maximum media awareness is an ideal measure in curbing alcohol abuse in the community. The findings of this study will provide useful insight to the government and policy makers in targeting suitable media awareness programs in combating alcoholism.

# DEDICATION

This work is dedicated to my lovely parents Mr and Mrs Muchika.

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# ABBREVIATION AND ACRONYMS

AEP	Alcohol Equilibrium Point
AFE	Alcohol Free Equilibrium
DFE	Disease Free Equilibrium
EEP	Endemic Equilibrium Point
MATLAB	Mathematical Laboratory
NACADA	National Authority for the Campaign against
	Alcohol and Drug Abuse
ODE	Ordinary Differential Equation
$R_0$	Reproduction number
WHO	World Health Organization

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#### CHAPTER 1

#### INTRODUCTION

#### 1.1 Background of the Study

The use of alcohol and other drugs has been seen as a persistent public health issue all across the world, notably in Africa [13]. There have been reports of disastrous repercussions on people, families, and societies as a result of rising alcohol and other drug usage. The World Health Organization [24], defines drug abuse as a collection of cognitive, behavioral, and physiological signs indicating that someone is heavily using psychoactive drugs, which can result in addiction syndrome. A study by Otieno [18], indicates that, alcohol, amphetamines, barbiturates, and opioids (codeine, cigarettes, heroin, morphine) are some of the substances that are commonly abused [24].

Drug abuse is linked to both short- and long-term impacts. The effects of drug usage vary depending on the drug or other substance being used, the user's medical history, and other factors. The short-term repercussions of substance addiction include intoxication resulting from alcohol consumption. It is possible that one may feel weak, uninhibited, or relaxed. Changes in cognitive function, change in eating habits, an elevated heart rate, and inadequate sleep are some other short-term consequences of drug abuse. On the other hand, some long-term consequences of drug abuse include adjustments in psychological functioning, which may include heightened aggression, memory loss, and attention problems. Cardiovascular problems may result from excessive drug usage. For instance, excessive use of alcohol harms the liver and causes cirrhosis. Drug abuse thus has an impact on a person's life and may go beyond their physical conditions. This includes; relationship problems, low academic progress, and withdrawal symptoms. [23].

According to NACADA's report [17], the causes of drug abuse include inadequate parental guidance, peer pressure, social media influence among others. A study by Zhang and Snizek [25] shows that drug abuse can lead to addiction or dependence. According to Armeli *et al.* [2], substance abuse shows a complex interplay between the user, the substance they are abusing, and society. Dependence involves physiological processes, whereas addiction entails a need to continue using the substance despite the consequences. According to a study by Pham [19], even though it is legal to consume substances like alcohol, cigarettes, prescription medications, or odors from home goods, when these substances are used inappropriately, even a small amount of them can change how the brain functions.

#### 1.1.1 Alcoholism

Alcohol is a narcotic that many people consume around the world and is addictive. Alcohol is taken in by small blood vessels found in the stomach walls and small intestine. It is then moved to the neural system where the brain is affected and slowed down. Depending on the amount of alcohol ingested, how frequently one drinks and occasionally the kind of alcohol drank, alcohol has serious effects on the body. If used sparingly, it has stimulating effects, while heavy use might have depressive consequences. Alcoholism can therefore be seen as a result of either social epidemics, personal preferences and family history [24].

Alcoholism is the misuse and binge drinking of alcohol, which can have

negative effects on everyone in the society on a physical, social, and moral level [24]. The World Health Organization has calculated that alcohol and drug abuse caused nearly 3 million deaths worldwide in the past year, this translates to 5.9% of all deaths. According to current estimates, 7.9 million people in the United Kingdom consume alcohol, up from 6.5 million the year 2021, or a 22 percent increase [23]. For the case of Africa, Rwanda has a 17.4% addiction prevalence and this corresponds to 76% of the entire population as shown by Milanzi [11]. NACADA stipulates that alcohol abuse is most prevalent in Kenya, with a prevalence rate of roughly 31%. About 60% of Kenyans have used alcohol, and nearly half of them have experienced negative consequences from consuming alcohol. Every year, alcohol misuse claims the lives of four out of every 100 Kenvans [17]. Based on these troubling statistics of alcohol consumption, interventions and prevention strategies have been developed to help those who are affected [17, 23]. Alcoholism has become a serious global challenge due to its social and economic ramifications on different strata of the community. Alcoholism has contributed to a large burden of chronic diseases and even increase in deaths as a result of these diseases and accidents [23, 24]. Therefore, various prevention and treatment strategies such as rehabilitation, guidance and counseling have been targeted to address the problems of the affected individuals worldwide. Furthermore, awareness has been emphasized to curb the spread and occurrence of alcoholism [13, 17, 23].

#### 1.1.2 Media Awareness

To lower the number of alcohol abuse, prevention and treatment techniques like rehabilitation and awareness campaigns have been addressed [23]. Approximately 70% of alcoholics who receive treatment are able to decrease the number of days they consume alcohol in excess, which improves their health within six months, according to a World Health Organization report [23]. Teenagers and adults need to be exposed to mass media programs because they raise awareness on the effects of alcohol abuse. Any method of information transmission to a large audience at once is referred to as mass media [16].

Evidence from studies such as [4, 16] shows that media awareness campaigns are a sensible strategy to educate the public about alcohol abuse. Digital media refers to any communication medium that uses different encoded machine-readable data formats. Digital media, print media, broadcast media, and outdoor media may all fall under mass media. Broadcast media involves electronically distributing information to a large number of receivers at once that includes signals, print messages, and audio or video material. Music, videos, television tutorials, and print media are all examples of broadcast media. Social media refers to a computer-based technology that makes it possible to share concepts, ideas, and knowledge with others via online groups and networks. Social media platforms like Instagram, Twitter, Whatsapp, and MySpace are examples of digital media. Print media is internet-based communication that includes newspapers, periodicals and journals. Outdoor media consists of placards, billboards in public places and augmented reality commercials. When effective campaigns are launched against alcohol abuse, such media outlets serve as important informational resources and not only change people's behavior but also raise the government's engagement in health care [16].

These behavioral reactions can reduce drinking habits and alter the ways that alcoholism spreads. As a result, it is necessary to include the impact of media awareness efforts in the mathematical models. The effect of media awareness campaigns on alcohol addiction has recently been highlighted by a number of mathematical models such as [4, 9, 16]. According to the findings of this studies, media outlets can significantly slow the growth of alcoholism. This is due to the rapid dissemination of information on the effects of alcoholism to a large audience.

#### 1.2 Mathematical Modeling

According to the studies such as [12, 13, 24] alcohol consumption is still a serious public health issue worldwide and is by no means under control. There is evidence that alcoholism spreads like an infectious disease, according to Misra [12], therefore it can be represented mathematically. A crucial tool for comprehending the emergence and spread of alcoholism throughout ages has been mathematical modeling. The real-world phenomenon of alcohol abuse is represented mathematically by using mathematical concepts, language, and symbols [14].

Numerous mathematical models have examined alcoholism in relation to how it is influenced by media awareness campaigns and treatment. However, there hasn't been much attention paid to how exposed class of alcoholics affect both light and heavy drinkers. Alcoholism has no known cure, although raising awareness, particularly through the media, may be essential to lowering the population of drinkers in a given area [9]. Motivated by the study by Muthuri *et al.* [16], a mathematical model of alcoholism incorporating awareness and the influence of exposed class on light and heavy drinkers is formulated and its long term solutions analyzed.

# 1.3 Statement of the Problem

Alcoholism has become a serious public health threat worldwide, notably in Africa [13, 23]. The high incidence of alcohol abuse in the continent suggests that the spread and severity of alcoholism continue to be rather high, even with the presence of treatment options like rehabilitation. About 60% adult Kenyans are rapidly engaging in alcoholism with nearly half of them suffering from adverse effects of abusing the drug [16, 17]. Four out of one hundred Kenyans loose their lives yearly due to alcohol abuse [17]. This can lead to low productivity of the country, social conflicts and even death. To address and curb the abuse of alcohol, media awareness should be directed towards the affected individuals [9, 16]. Mathematical models on alcoholism incorporating treatment have been formulated. The models have assumed that after successful treatment individuals quit alcohol, rather than considering the fact that some may become susceptible again [6, 9, 16]. However, the influence of the exposed class of alcoholics on light and heavy drinkers in the presence of media awareness throughout the alcoholism process, has not been addressed. The exposed class is critical as it would aid in understanding the spread, nature and extent of alcoholism at the population level. Thus, a mathematical model of alcoholism that incorporated the influence of the exposed class on light and heavy drinkers in presence of media awareness was formulated and analyzed.

#### 1.4 Objectives of the Study

### 1.4.1 Main Objective

The main objective of this study is to formulate and analyze a mathematical model of alcoholism incorporating media awareness as an intervention strategy and the influence of the exposed class on light and heavy drinkers.

# 1.4.2 Specific Objectives

The specific objectives of the study are;

- To formulate a mathematical model of alcoholism incorporating media awareness as an intervention strategy.
- (ii) To perform stability analysis of the model at each equilibrium point using the Jacobian matrix method.
- (iii) To perform numerical simulations of the model using MATLAB software, where secondary data from NACADA and data from Kenya rehabilitation centers is used to validate the analytical results of the impact of media awareness on alcoholism.

## 1.5 Justification of Study

This study was inspired by the study done by Muthuri *et al.* [16], which demonstrates the effectiveness of media awareness and treatment programs in reducing alcohol abuse in a community. Furthermore, the study is motivated by the study done by Ma *et al.* [9], which demonstrates how awareness campaigns can reduce the heavy drinkers' population. Alcoholism is a growing hazard to global public health, particularly in Africa. It is a significant societal issue to research due to the rise in fatalities and the catastrophic effects it has on all spheres of human life. Alcohol abuse accounts for about 5.9% of the global fatalities and about 5.1% of the chronic conditions such as alcoholic liver, cirrhosis among others [24]. Even though there are programs to prevent and treat alcoholism, the prevalence of alcohol use is still quite high as reported by the World Health Organization [23]. Additionally, despite the creation of numerous mathematical models of alcoholism that incorporate rehabilitation, the tendencies of alcohol addiction continue to be difficult to predict. This might be due to poor implementation of the intervention measures which has led to the rising burden of chronic illnesses such as cirrhosis and a large cost to the economy. Therefore, there is need to target prevention and treatment measures that will account for the serious repercussions of alcoholism. Thus the findings of this study will inform the government and other policy makes on the need to target media awareness with more emphasis on the exposed class in order to minimize alcoholism cases.

#### 1.6 Significance of Study

Understanding the occurrence, nature and spread of alcoholism requires mathematical modeling. The exposed class and media literacy have been taken into account in the study. The exposed class is intended to facilitate further minimizing of light and heavy drinkers in media awareness since it determines the force of alcohol actions. Policy makers and stakeholders like the government of Kenya and NACADA can benefit from the model's findings. That is, in terms of selecting effective media control tactics, planning, and allocating funds for media resources to raise public awareness.

This study will also provide information to higher education institutions so they can adopt media literacy initiatives like posters advertising products and outdoor events like plays about alcoholism and drug in general. As a result, there will be fewer incidences of alcoholism among students since they will be made more aware of the risks. The findings of this study will build knowledge and support existing knowledge by ascertaining whether mathematical ideas on alcoholism are supported by the study or if they still need further proof before they can be considered as knowledge. Researchers in academic and medical institutions are continually examining a vast array of subjects, including alcoholism. As mathematical modeling theories go through the scientific process repeatedly, they are put to the test and then put to the test again, increasing the confidence in them. It is possible to adapt, broaden, and combine alcoholism ideas to create increasingly convincing explanations because to this iterative process.

The results of this study will pave the way for a more cooperative effort in which all mathematicians, scientists, and the general public will cooperate to assist address the problem of alcohol consumption in the ever-evolving globe. This entails conducting more study on the consequences of alcoholism on all aspects of human life and boosting media campaigns by creating new technologies and making educated decisions on the subject of alcohol misuse at the local, national, and international levels. Finally, the study will help the society in promoting a drug-free society and assuring social, economic, and political advancement in line with sustainable development goals.

#### CHAPTER 2

#### LITERATURE REVIEW

#### 2.1 Recovery Models for Alcoholism

Walters *et al.* [21] developed a mathematical model to explain alcoholism. According to the study, there are three primary categories of drinkers: susceptible drinkers, denoted by S, who occasionally consume alcohol but later develop drinking problems, addicted individuals, denoted by A, and those in recovery, denoted by R. The following sets of linear equations were applied in the study:

$$S' = \mu N - \beta \frac{A}{N} S - \mu S$$
  

$$A' = \beta \frac{S}{N} A + P \frac{R}{N} A - (\mu + \phi) A$$
  

$$R' = \phi A - P \frac{A}{N} R - \mu R$$
(2.1)

The model's parameters are listed in [21]. By extracting the Jacobian matrix's eigenvalues, the reproduction number  $R_0$ , was calculated. Analysis showed that, alcohol remained in the population when  $R_0 > 1$ , and it disappeared when  $R_0 < 1$ . Sensitivity analysis revealed that the reproduction rate was significantly impacted by  $\beta$  (rate of susceptibility acquiring alcohol issue), which was based on contact with the drinking class. The reproduction number increased when  $\beta$  increased while it decreased when  $\beta$ decreased. The study came to the conclusion that keeping the susceptible population from joining the alcohol problem class lowers the population's alcohol problem. Since the primary goal is to reduce addictions in the community, the fact that this strategy addressed the need to recover from addiction is encouraging. The study, however, did not specifically highlight the impact of exposed class on light and heavy drinkers and did not offer a clear treatment plan such as media awareness for addressing the alcohol problem.

A model developed by Ander *et al.* [1] investigated how community drinking patterns were changed by media awareness campaigns. A four compartmental model with compartments; S(t) which denoted the susceptible people who drink lightly or moderately, A(t) which denoted problem drinkers with heavy drinking habits, R(t) which denoted people in treatment for alcoholism or who have just recovered from it, and M(t) which denoted density of media. The following differential equations were utilized;

$$\frac{dS}{dt} = \mu - \beta SA - \mu S,$$

$$\frac{dA}{dt} = -\lambda AM + \alpha AR + \beta SA - \mu A,$$

$$\frac{dR}{dt} = \lambda AM - \alpha AR + \mu R,$$

$$\frac{dM}{dt} = \gamma_1 A - \gamma_2 M.$$
(2.2)

The parameters of the model have been described in [1]. According to the stability study of the model, the system was stable when  $R_0 < 1$ , and unstable when  $R_0 > 1$ . When  $R_0 > 1$ , a unique endemic state existed. This has a practical implication that whereas there were no cases of alcohol abuse in the first scenario, more alcohol users were anticipated in the second, necessitating the establishment of a suitable management approach to handle the escalating alcoholics. As per this study, the basic reproduction number was influenced by both the extent of problematic drinking's peer pressure on susceptibles and the average amount of time spent in a problem drinking becomes ingrained in the culture and can only be prevented by

lowering the transmission rate  $\beta$  or the amount of time spent in a drinking environment,  $\frac{1}{\mu}$ . According to the study's findings, if drinking culture has already developed, prevention strategies like strong media awareness campaigns and thoughtfully crafted treatment choices will only assist contain or mitigate the growth of problem drinking, not completely eradicate it. The incorporation of the problematic drinking class boosted this model since its a critical class that is often assumed by most models. Nevertheless, the impact of exposed class on light and heavy drinkers was disregarded despite the model's simplicity for better understanding. Consequently, without the exposed individuals and media awareness incorporated, the spread of alcohol may not be easily identified and minimized .

#### 2.2 Optimal Control Method Model of Drug Abuse

A nonlinear mathematical model was developed by Wang *et al.* [22], to account for the impact of awareness campaigns on binge drinking. The model made use of a SATQ methodology and had compartments for susceptibility, alcohol abuse, treatment, and quitting. By introducing an objective functional and applying Pontryagin's Maximum Principle, the best control techniques are determined. That is, the study reexamined the system and employed two control factors to decrease the number of alcoholics in order to investigate an effective campaign to control alcoholism by pursuing the goals of the decreased alcoholism and more recovered individuals. The following nonlinear ordinary differential equations were utilized in the study:

$$S' = \mu N - \mu S - \frac{1 - \mu_1 \beta S A}{N},$$
  

$$A' = (1 - \mu_1) \frac{\beta S A}{N} + \xi T - (\mu_2 + \mu) A$$
  

$$T' = \mu_2 A - (\mu + \xi + \delta) T,$$

$$Q' = \delta T - \mu Q \tag{2.3}$$

The model's parameters are discussed in [22]. The AFE's stability requirements were used to calculate the reproduction number, and analysis revealed that there was an alcohol-free equilibrium which was locally asymptotically stable whenever  $R_0 < 1$  (no occurrence of alcoholism). Alcoholism equilibrium existed whenever  $R_0 > 1$  and was globally asymptotically stable, (alcoholism persisted in the population). Further analysis of the model found that optimal control has a greater impact than other control methods, indicated by a rise in the number of susceptibles and a decline in the number of alcohol users. Numerical simulation results revealed that awareness campaigns are helpful in lowering alcohol-related disorders. The model has a strength because it includes an optimum control method, in which dynamical systems are tuned over time to minimize alcohol misuse while optimizing an objective function. However, the study did not include the influence of the exposed class of alcoholism on light and heavy drinkers in presence of awareness programs. Thus, without the influence of exposed individuals in media awareness programs, treatment may not be quite effective for individuals to quit alcoholism.

Manthey *et al.* [10] utilized a simple mathematical model to illustrate the dynamics of college students' alcohol-use patterns. The study divided the student population into three groups: those who don't drink (N), those who drink socially (S), and those who drink problematically (P). In order to establish the dynamics of on-campus drinking, the following sets of nonlinear differential equations were used:

$$\frac{dN}{dt} = \eta - \eta N - \alpha NS - kNP + \beta S + \varepsilon P,$$
  
$$\frac{dS}{dt} = \sigma S - (\eta + \sigma)S + \alpha NS - \beta S - \gamma SP + \delta P,$$

$$\frac{dP}{dt} = \pi P - (\eta + \pi)P + \gamma SP + kNP - \delta P - \varepsilon P, \qquad (2.4)$$

The model's parameters are listed in [10]. The local stability was determined using the eigenvalues of the Jacobian matrix. Analysis showed that the AFE point, was stable whenever  $R_0^s < 1$ , implying that there was no drinking culture in campuses. The endemic equilibrium existed whenever  $R_0^s > 1$  and was stable (increase in drinking habits). Numerical simulation was carried out using MATLAB, and analysis of the findings revealed that, in order to lessen addiction, recruitment patterns among educational institutions must be changed. The study specifically suggested that to reduce alcohol and other drug use, the capacity of problem drinkers to solicit non-drinkers directly should be restricted. The study included different categories of drinkers, such as social drinkers and problem drinkers, which other studies hardly ever take into account. The impact of exposed class on problem drinkers as well as media awareness makes it challenging to help lessen the number of alcohol-related problems.

A study by Mushayabasa in [15], formulated an S I  $I_{\alpha}MR$  model of drug abuse. This model contained pertinent social and biological components, accounted for the cases of drug addicts, and allowed for the employment of the best control measures. The susceptible class is denoted by S, the occasional or light drug usage is denoted by I, strong drug use was denoted by  $I_{\alpha}$ , people with mental illness were denoted by M, and detected drug users were denoted by R. The study employed the following sets of differential equations:

$$S' = \mu - \lambda S - \mu S$$
$$I' = \lambda S - (\alpha + \gamma + \sigma + \mu + \Psi)I$$

$$I'_{\alpha} = \alpha I - (\rho + \Phi + \mu + d)I_{\alpha}$$
  

$$M' = \sigma I + \Phi I_{\alpha} - (\varepsilon + \mu + \delta)M$$
  

$$R' = \gamma I + \rho I_{\alpha} + \varepsilon M - (\mu + \omega)R$$
(2.5)

The parameters of the model have been presented in [15]. Global stability was performed on both the DFE point and the drug persistent equilibrium by utilizing the Lyapunov function. The analysis of the model demonstrated that the system was globally asymptotically stable at the drug-free equilibrium whenever  $R_d < 1$  and that there was a specific drug persistent equilibrium that was globally asymptotically stable whenever  $R_d > 1$ . Simulations performed using MATLAB indicated that drug use among individuals within the community be it in an institution or people within one region are influenced by susceptible population. Incorporating optimal control methods to the social and biological aspects makes this model advantageous. However, this study did not take into account the influence of exposed group of drug users on light and heavy drinkers in presence of media awareness. Thus without considering the influence of the exposed group in alcoholism process, more individuals become susceptible to alcohol and may be addicted in the course of time due to rampant usage of alcohol.

# 2.3 Alcohol Models with Cumulative Density of Media Campaigns

Misra *et al.* [12] incorporated media cumulative density of awareness efforts into the model, which is a different dynamic variable whose growth is based on the magnitude of the epidemic. The following sets of nonlinear differential equations were examined:

$$\frac{dX}{dt} = A - \beta XY - \lambda XM - dX + \nu Y + \lambda_0 X_m$$

$$\frac{dY}{dt} = \beta XY - \nu Y - \alpha Y - dY$$

$$\frac{dX_m}{dt} = \lambda XM - dX_m - \lambda_\alpha X_m,$$

$$\frac{dM}{dt} = \mu Y - \mu_0 M$$
(2.6)

Parameters of the model have been presented in [12]. Stability conditions of the DFE and EE were determined. Stability conditions of the DFE were used to determine the basic reproduction number. Analysis showed that the DFE was stable whenever  $R_0 < 1$  (no infections in the population) and unstable whenever  $R_0 > 1$ . Alternatively, the endemic equilibrium was present and locally stable anytime  $R_0 > 1$ . This implied that more infections were anticipated, necessitating the need to enhance awareness campaigns in order to lower the high use of alcohol. Numerical results showed that media awareness campaigns are carried out in proportion to the number of the infected population. Taking into account how media influences a population that is prone to be confined, the study came to the conclusion that media campaigns designed to raise public awareness can help stop the spread of infectious conditions by separating some susceptible people from other infected individuals. This model benefits from the inclusion of the cumulative density of awareness initiatives since it is crucial for characterizing the probability distribution of the model's random variables. However, the study disregarded the impact of exposed class on light and heavy drinkers. Huo and Wang [7], developed a model of binge drinking under the effect of media coverage without taking recruitment and death into account. Depending on the degree of the nodes, they split the population N into n groups (n being the maximum degree), and then these groups were separated into the following classes according to how they consumed or used alcohol; the number of nondrinkers or light drinkers whose degree is k was represented

by  $S_k(t)$ . The number of awareness individuals who shield away from binge drinkers were represented by  $X_k(t)$ . The proportion of heavy drinkers was given by  $I_k(t)$ , and total density of media-driven awareness campaigns was given by M(t). Using mean-field theory, they created a model for binge drinking that took the media into account. The following 3n + 1 system of ordinary differential equations was employed.

$$\frac{dS_k(t)}{dt} = -\beta k S_k \theta(t) - \alpha S_k M + \mu I_k + \sigma X_k,$$

$$\frac{dX_k(t)}{dt} = \alpha S_k M - \sigma X_k,$$

$$\frac{dI_k(t)}{dt} = \beta k S_k \theta(t) - \mu I_k, k = 1, 2, ..., n,$$

$$\frac{dM(t)}{dt} = \omega \sum_{k=1}^n I_k - \gamma M.$$
(2.7)

The parameters of the model have been presented in [7]. After analysis, they showed that the alcohol-free equilibrium points  $E_0$  was locally asymptotically stable whenever  $R_0 < 1$  and became unstable whenever  $R_0 > 1$ . Additionally, the unique alcohol equilibrium existed and was globally asymptotically stable when  $R_0 > 1$ . Additionally, when

 $R_0 > 1$ , the unique alcohol equilibrium existed and was globally asymptotically stable. The practical implication of this was that, when the reproduction number was less than one, alcohol addiction in the population vanished, and when the reproduction number was more than one, it persisted. Simulation results indicated that heterogeneity of the network made drinking behaviour to spread faster. Thus, the study came to the conclusion that mixing freely makes drinking behavior spread rapidly and that media coverage is the greatest strategy to alleviate alcohol problems even though it does not change how it spreads. The inclusion of degree of distribution as

the maximum value for media coverage boosted this model. Nevertheless, the influence of the exposed class on light and heavy drinkers' was omitted in the model.

Studies by Dubey [6] developed a SIR model that examined the effects of two crucial factors: awareness campaigns and treatment, on the transmission of infectious conditions. The study separated the population into the following classes: susceptible individuals denoted by S, infectious individuals denoted by I, and recovered individuals denoted by R. Then it was supposed that a subset of the S class makes up a class known as the susceptible awareness population, abbreviated as  $S_a$ . It develops from exposure to awareness campaigns by social and electronic media density denoted by M. The following sets of differential equations were employed in the study:

$$\frac{dS}{dt} = A - \delta_0 S - \alpha SI - \frac{\beta}{S} M1 + \gamma M + \delta_3 S_a$$

$$\frac{dI}{dt} = \alpha SI - \delta_0 I - \delta_1 I - \delta_2 I - \frac{aI}{1 + bI}$$

$$\frac{dS_a}{dt} = \frac{\beta SM}{1 + \gamma M} - \delta_0 \delta_a - \delta_3 \delta_a$$

$$\frac{dM}{dt} = \mu I - \mu_0 M$$

$$\frac{dR}{dt} = \delta_2 I - \delta_0 R + \frac{aI}{1 + bI}$$
(2.8)

The model's parameters are listed in [6]. The study's conclusions indicate that there are two equilibrium points: DFE (total eradication of the virus, when I = 0), and endemic equilibrium (the spread of the disease increased). When  $R_0 < 1$  the system was globally asymptotically stable, endemic equilibrium was predicted whenever  $R_0 > 1$ . The simulations were run using MATLAB, and the findings indicated that when the media awareness rate,  $\beta$ , increased, the infected population declined. The investigation came to the conclusion that a lack of media distribution increased infection, which was further decreased by treatment. The incorporation of the susceptible awareness class boosts this model since the rate of awareness is high hence further minimization of the drinking population. However, the study neglected the influence of the exposed class of alcoholism and efforts to minimize it through awareness. Without emphasis on the influence of exposed class in alcoholism process, reduction of the infection may be relatively problematic.

#### 2.4 Pre-Exposure to Media Campaigns Model

Muthuri *et al.* [16], developed a mathematical model to predict how treatment and targeted mass media efforts will affect alcohol abuse in Kenya. The model was of the form  $SS_{\alpha}LHTQM$ , the susceptible individuals denoted by S, individuals exposed to media campaigns denoted by  $S_{\alpha}$ , the light drinkers denoted by L, the heavy drinkers denoted by H, individuals receiving treatment or in rehab facilities denoted by T, and Q denoted individuals who had permanently stopped drinking. The volume of antialcohol media efforts is measured by M. Two models were developed, one having successful pre-exposure marketing and the other failing. The study employed the following sets of linear equations:

$$\begin{aligned} \frac{dS}{dt} &= \Lambda + \omega S_a - \beta_m SM - (\lambda + \mu)S \\ \frac{dS_a}{dt} &= \beta_m SM - (\omega + \mu)S_a - \lambda(1 - \varepsilon)S_a \\ \frac{dL}{dt} &= \lambda S + \lambda(1 - \varepsilon)S_a - (\mu + \sigma_1 + \sigma_2)L \\ \frac{dH}{dt} &= \alpha_1 L + \tau_2 t - (\mu + \sigma_1 + \sigma_2 + \delta)H, \\ \frac{dT}{dt} &= \sigma_1 H - (\mu + \tau_1 + \tau_2)T \\ \frac{dQ}{dt} &= \alpha_2 + \alpha_2 H + \tau_1 T - \mu Q \end{aligned}$$

$$\frac{dM}{dt} = \Theta_1 L + \Theta_2 H - \rho M \tag{2.9}$$

The model's parameters are discussed in [16]. The next generation matrix was used to calculate the alcohol reproduction number. The stability analysis of the model showed that the AFE point is locally asymptotically stable when  $R_0 < 1$  and unstable when  $R_0 > 1$  (meaning that there are no cases of alcoholism in the society). Further research shows that the EE point occurs and is positive whenever  $R_0 > 1$  (increase in the number of heavy drinkers in the population thus necessity for treatment so as to reduce the drinking endemic).

According to numerical simulations carried out with MATLAB, increasing the rate of treatment reduces the population of alcohol addicts, and positive media campaigns reduce alcohol consumption. According to the analysis, the most efficient way to lower the population of heavy drinkers is by treatment. The complex nature of the model necessitated the estimation of many variables. The study did not consider the influence of exposed individuals on light and heavy drinkers. The study assumed that an individual only comes into contact with a light drinker for him/her to become an alcoholic, and yet heavy drinkers also contribute to infections of alcoholism. The study also assumed that after treatment individuals only quit alcohol instead of becoming susceptible again.

Model	Strength	Weakness/Gap	Ref.
Modeling alco- hol problems: Total recovery.	The model addressed the need to recover from addic- tion hence a boost since the main aim is to minimize ad- dictions in the community.	The study was limited to ad- dictions and recoveries from al- cohol but not media aware- ness and the influence of ex- posed class on light and heavy drinkers.	[21]
Alcoholism: A mathematical model with media awareness campaigns.	The model examined how media awareness efforts af- fected the prevalence and persistence of drinking in a community.	The incorporation of the prob- lematic drinking class boosted this model as it is a criti- cal class in the study of alco- holism. However the influence of the exposed class on light and heavy drinkers was omit- ted.	[1]
Optimal Control Strategies in an Alcoholism Model.	The model incorporated op- timal control strategy where dynamical systems over a period of time are such that an objective function is op- timized, hence further min- imization of alcohol abuse	However, the study did not emphasize the influence of the exposed class of alcoholism on other groups of drinkers in presence of media awareness programs.	[22]
Campus drink- ing: an epi- demiological model.	The study considered other unique drinking classes such as social and problem drinkers.	The study failed to address the impact of media awareness on alcoholism and the influence of exposed class on light and heavy drinkers.	[10]
Modeling illicit drug use dynam- ics and its opti- mal control anal- ysis	Incorporating optimal con- trol methods to the so- cial and biological aspects makes this model advanta- geous.	Despite the incorporation of optimal control methods, the study did not address the in- fluence of exposed class of drug users on light and heavy drinkers in presence of media awareness.	[15]
Modeling and analysis of effects of aware- ness programs by media on the spread of infectious diseases.	The study considered cu- mulative density function of media awareness programs which is essential in describ- ing the probability distribu- tion of random variables.	Although, the study consid- ered cumulative density func- tion of media awareness pro- grams which is essential in de- scribing the probability dis- tribution of random variables, the influence of exposed class was not examined.	[12]

# 2.5 Summary of the Alcoholism Models

Model	Strength	Weakness/Gap	Ref.
Impact of media	The model emphasized the	The model took into con-	[7]
coverage on the	effect of media coverage	sideration the degree of dis-	
drinking dynam-	without taking recruitment	tribution of media coverage	
ics in the scale-	and death into account.	but ignored the influence of	
free network		the exposed class on light	
		and heavy drinkers'.	
Role of media	The model was simple hence	Despite the model being	[6]
and treatment	a better understanding and	simple for better under-	
on an SIR	the incorporation of the	standing, and the incor-	
model.	susceptible awareness class	poration of the susceptible	
	boosts this model since the	to explore the influence of	
	high honco further mini	avposed class on light and	
	might hence further mini-	heavy drinkers	
	population.	nearly armitere.	
Modeling the ef-	The study considered two	However, the the influ-	[16]
fects of targeted	models in assessing the im-	ence of exposed individ-	
mass media	pact of media awareness	uals on light and heavy	
campaigns and	hence its complexity nature	drinkers was not addressed,	
treatment on	necessitated the estimation	the study assumed that	
alcohol abuse in	of many variables.	an individual only comes	
Kenya.		into contact with a light	
		drinker for him/her to be-	
		come an alcoholic, and yet	
		heavy drinkers also con-	
		tribute to infections of alco-	
		sumed that after treatment	
		individuals only quit alcohol	
		instead of becoming suscep-	
		motoria or percomme publicity	

# 2.6 Research Gaps

Several mathematical models for alcoholism such as [6, 9, 16] have been formulated. media wareness has also been captured in some of these models as an intervention/treatment strategy. Most of these models failed to consider the fact that after successful media awareness, some individuals become susceptible again. However, the influence of the exposed class of alcoholics on light and heavy drinkers in the presence of media awareness throughout the alcoholism process, has not been addressed. The exposed class of alcoholics is vital in determining the spread and extent of alcoholism. This may ensure effective minimization of the drinking population. By examining a mathematical model of alcoholism that incorporates media awareness and the impact of the exposed alcoholic class on light and heavy drinkers, the study closed the knowledge gap. As a result, this research will help in understanding how media awareness campaigns might reduce the drinking population and help combat alcoholism.

#### CHAPTER 3

# FORMULATION AND ANALYSIS OF THE MODEL

#### 3.1 Model Formulation

The model was formulated based on a system of differential equation. The model has five compartments (classes) considering the entire population. These classes are; the susceptible individuals who either have or have never consumed alcohol in their lifetime and can also result from media awareness individuals and denoted by S. The exposed class denoted by E, comprise of individuals who are at risk of becoming alcoholics as a result of contact with light and heavy drinkers. The light drinkers denoted by  $V_1$ , are individuals who drink occasionally and can do with or without alcohol. The heavy drinkers denoted by  $V_2$ , are individuals who are highly addicted to alcohol or rather dependent on alcohol. The media awareness class denoted by A, are individuals undergoing media awareness.

Five ordinary differential equations were used to represent the change from one compartment to another, as shown in figure 3.1. Individuals are brought into the model at a rate of  $\Lambda$ . The rate of progression from S to E is given by  $\omega$ , the progression rate to  $V_1$  from E is  $\alpha_1$ , the rate of progression from  $V_1$ to  $V_2$  is given by  $\alpha_2$  and the progression rate from  $V_2$  to  $V_1$  is  $\kappa$ . Fatalities from other causes occur at the rate of  $\mu$  and alcohol-related causes at a rate of  $\sigma$ .  $\tau_1$  and  $\tau_2$  represent the rates at which light and heavy drinkers transit to the media awareness campaigns, respectively. The rate at which an individual becomes susceptible again after undergoing media awareness is given by  $\beta$ .

# 3.1.1 Model Assumptions

The study was based on the following assumptions.

- (i) Alcohol addicts must be exposed to media awareness if they want to stop drinking because they cannot recover on their own by self-control.
- (ii) Not all individuals will quit alcohol completely.
- (iii) After undergoing media awareness, an individual can get back to being susceptible.
- (iv) The exposed class results from contact with both light and heavy drinkers.
- (v) Individuals in the light drinking class drink occasionally and can do with or without alcohol.

## 3.1.2 Model Flow Chart and Equations

The model below summarizes the variables and parameters described in section 3.1.



#### Fig 3.1: Schematic Diagram of the Proposed Model

The following sets of equations governed the model:

$$\frac{dS}{dt} = \Lambda + \beta A - \mu S - \omega S(V_1 + V_2) 
\frac{dE}{dt} = \omega S(V_1 + V_2) - \mu E - \alpha_1 E 
\frac{dV_1}{dt} = \alpha_1 E + \kappa V_2 - (\alpha_2 + \tau_1 + \mu + \sigma_1) V_1 
\frac{dV_2}{dt} = \alpha_2 V_1 - (\kappa + \tau_2 + \mu + \sigma_2) V_2 
\frac{dA}{dt} = \tau_1 V_1 + \tau_2 V_2 - (\mu + \beta) A$$
(3.1)

By non-dimensionalization, let;

 $k_1 = \mu + \beta, k_2 = \alpha_2 + \tau_1 + \mu + \sigma_1, k_3 = \kappa + \tau_2 + \mu + \sigma_2.$ 

# 3.2 Positivity and Boundedness of the Model

The components to be examined are; the invariant region, boundedness and the model's positivity.

# 3.2.1 Invariant Region

**Lemma 3.1.** All feasible solutions of the system in equation (3.1) are bounded and enter into the following region;

$$\Omega = S(t), E(t), V_1(t), V_2(t), A(t) \in \mathbb{R}^5_+$$

*Proof.* If  $(S, E, V_1, V_2, A)$  is a solution to the system in equation (3.1) with non-negative initial conditions, summing the five equations i.e  $N = S + E + V_1 + V_2 + A$  the following solution is obtained;

$$\frac{dN}{dt} \leq \Lambda - \mu N - \sigma_1 V_1 - \sigma_2 V_2 \tag{3.2}$$

In absence of mortality rate due to alcohol, equation (3.2) reduces to;

$$\frac{dN}{dt} \leq \Lambda - \mu N \tag{3.3}$$

Integrating equation (3.3) with respect to time using the integrating factor method. The equation is first re-written in standard form as;

$$\frac{dN}{dt} + \mu N \le \Lambda$$

the integrating factor is given by,

$$\exp^{\int \mu dt} = \exp^{\mu t}$$

The left hand side of equation is multiplied by the integrating factor and re-written as,

$$\frac{d}{dt}(\exp^{\mu t}.\mu N) \le \Lambda$$

Integrating this equation yields;

$$N(t) \leqslant \frac{\Lambda}{\mu} \tag{3.4}$$

It is evident from equation (3.4) that N(t) is bounded and

 $0 \leq N(t) \leq \frac{\Lambda}{\mu} + N(0) \exp^{\mu t}$ . Boundedness implies that there is no growth beyond limit due to unsustainable resources.

Where N(0) serves as the initial value of the total population. If N(0) >  $\frac{\Lambda}{\mu}$ , then the solution enter  $\Omega$  in finite time or N(t) approaches  $\frac{\Lambda}{\mu}$  asymptotically. The investigation of the study shows that the feasible solutions set of the system equations enters and remain in the region  $\Omega$  for all future time, where;

$$\Omega = (S, E, V_1, V_2, A) \in \mathbb{R}^5_+ \mid 0 \le N(t) \le \frac{\Lambda}{\mu}$$

as  $t \longrightarrow \infty$ .

As a result, the model is well posed from equation (3.4), and the dynamics of alcohol abuse in the model may be examined in  $\Omega$ . That is the population growth is always bounded by  $\frac{\Lambda}{\mu}$ .

# 3.2.2 Positivity of the Model Solutions

**Lemma 3.2.** If the initial values S(0), E(0),  $V_1(0)$ ,  $V_2(0)$  and A(0) are positive, then the system in equation (3.1) has positive solutions of S(t), E(t),  $V_1(t)$ ,  $V_2(t)$ , A(t)for all t > 0.

*Proof.* Assuming the initial conditions are as follows;

 $S(0) > 0, E(0) > 0, V_1(0) > 0, V_2(0) > 0, A(0) > 0$ . Then from the first equation of the system in equation (3.1);

$$\frac{dS}{dt} \geq -\mu S - \omega S(V_1 + V_2) \tag{3.5}$$

Separating variables and integrating equation (3.5) with respect to time t;

$$\int \frac{dS}{S} \geq -\int (\mu + \omega(V_1 + V_2))dt \qquad (3.6)$$

The solution obtained from equation (3.6) is;  $\ln S \ge -(\mu + \omega(V_1 + V_2))t + C$ . Introducing exponents, the solution obtained is:

$$S(t) \ge S(0) \exp^{-(\mu + \omega(V_1 + V_2))t} > 0$$
 (3.7)

As a result, equation (3.5) is positive regardless of time t. The same procedure applies to differential equations involving E,  $V_1$ ,  $V_2$  and A and the solutions obtained are;

$$E(t) \ge E(0) \exp^{-(\mu + \alpha_1)t} > 0$$
 (3.8)

$$V_1(t) \ge V_1(0) \exp^{-k_2 t} > 0$$
 (3.9)

$$V_2(t) \ge V_2(0) \exp^{-k_3 t} > 0$$
 (3.10)

$$A(t) \ge A(0) \exp^{-(\beta + \mu)t} > 0$$
 (3.11)

Equations (3.8), (3.9), (3.10) and (3.11) are always positive for all time t. This shows that any instant t < 0, the population is positive (there is population growth). Therefore, the system of equations in model (3.1) are all positive for future time t and thus the system is biologically and mathematically well posed.

#### 3.3 Model Analysis

This section examines the Alcohol Free Equilibrium, alcohol reproduction number, local stability of AFE, and local stability of the endemic equilibrium.

## 3.3.1 Alcohol Free Equilibrium

All drinking classes and media awareness classes are set to zero to derive the AFE of the system in equation (3.1), that is  $E = V_1 = V_2 = A = 0$  and  $S \neq 0$ . Hence, the following is obtained.

$$S^0 = \frac{\Lambda}{\mu}$$

The AFE of the model is given by:

$$E^{0} = [S^{0}, E^{0}, V_{1}^{0}, V_{2}^{0}, A^{0}] = [\frac{\Lambda}{\mu}, 0, 0, 0, 0]$$

This means that there is no occurrence of alcoholism in the society and population growth will be described by the susceptible class and hence as  $t \longrightarrow \infty, S \longrightarrow \frac{\Lambda}{\mu}$ .

## **3.3.2** The Alcohol Reproduction Number $R_0$

That is the average number of secondary alcohol cases produced by one alcohol user throughout the alcoholism period. The alcohol reproduction number,  $R_0$ , is determined by use of the next generation matrix approach, which was employed by Catillo-Chavez et al., 2002 [3]. In most models, the initial infection is denoted by the letter F, and the transfer of infection with

the letter V, observing that  $S^0 = \frac{\Lambda}{\mu}$ . From the system of equation (3.1), new alcohol cases are given by  $\omega S(V_1 + V_2)$ , which can be written in matrix form as;

$$F = \begin{bmatrix} \omega S(V_1 + V_2) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

V is obtained by the transfer of equations out and into the model, as shown in the following matrix:

$$V = V_i^{-} - V_i^{+} = \begin{bmatrix} \frac{dE}{dt} = (\mu + \alpha_1)E \\ \frac{dV_1}{dt} = (\mu + \tau_1 + \alpha_2 + \sigma_1)V_1 - \kappa V_2 - \alpha_1 E \\ \frac{dV_2}{dt} = (\mu + \sigma_2 + \kappa + \tau_2)V_2 - \alpha_2 V_1 \\ \frac{dS}{dt} = (\mu + \omega E)S - \beta A - \Lambda \\ \frac{dA}{dt} = (\mu + \beta)A - \tau_1 V_1 + \tau_2 V_2 \end{bmatrix}$$

Differentiating equation F and V with respect to E,  $V_1$  and  $V_2$ , the following results are obtained;

$$F = \begin{bmatrix} \frac{\partial F_1}{\partial E} & \frac{\partial F_1}{\partial V_1} & \frac{\partial F_1}{\partial V_2} \\ \frac{\partial F_2}{\partial E} & \frac{\partial F_2}{\partial V_1} & \frac{\partial F_2}{\partial V_2} \\ \frac{\partial F_3}{\partial E} & \frac{\partial F_3}{\partial V_1} & \frac{\partial F_3}{\partial V_2} \end{bmatrix} = \begin{bmatrix} 0 & \omega S^* & \omega S^* \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$V = \begin{bmatrix} \frac{\partial V_1}{\partial E} & \frac{\partial V_1}{\partial V_1} & \frac{\partial V_1}{\partial V_2} \\ \frac{\partial V_2}{\partial E} & \frac{\partial V_2}{\partial V_1} & \frac{\partial V_2}{\partial V_2} \\ \frac{\partial V_3}{\partial E} & \frac{\partial V_3}{\partial V_1} & \frac{\partial V_3}{\partial V_2} \end{bmatrix} = \begin{bmatrix} \mu + \alpha_1 & 0 & 0 \\ -\alpha_1 & k_2 & -k \\ 0 & -\alpha_2 & k_3 \end{bmatrix}$$
(3.12)

The co-factors of matrix V in equation (3.12) are obtained to form another new matrix;

$$V = \begin{bmatrix} k_2 k_3 - \kappa \alpha_2 & \alpha_1 k_3 & \alpha_1 \alpha_2 \\ 0 & -k_3 (\mu + \alpha_1) & -\alpha_2 (\mu + \alpha_1) \\ 0 & \kappa (\mu + \alpha_1) & k_2 (\mu + \alpha_1) \end{bmatrix}$$
(3.13)

and the transpose obtained from equation (3.13) is given by;

$$V^{T} = \begin{bmatrix} -(k_{2}k_{3} - \kappa\alpha_{2}) & 0 & 0\\ -\alpha_{1}k_{3} & k_{3}(\mu + \alpha_{1}) & -\kappa(\mu + \alpha_{1})\\ -\alpha_{1}\alpha_{2} & \alpha_{2}(\mu + \alpha_{1}) & -k_{2}(\mu + \alpha_{1}) \end{bmatrix}$$
(3.14)

To obtain  $V^{-1}$ , the transpose is multiplied by the reciprocal of the determinant as follows;

$$V^{-1} = \frac{1}{(\mu + \alpha_1)(k_2k_3 - \kappa\alpha_2)} \begin{bmatrix} a_1 & 0 & 0\\ -\alpha_1k_3 & a_2 & a_3\\ -\alpha_1\alpha_2 & a_4 & a_5 \end{bmatrix}$$
(3.15)

where,  $a_1 = -(k_2k_3 - \kappa\alpha_2), a_2 = k_3(\mu + \alpha_1), a_3 = -\kappa(\mu + \alpha_1),$  $a_4 = \alpha_2(\mu + \alpha_1), a_5 = -k_2(\mu + \alpha_1).$  Multiplying F in equation (3.12) with  $V^{-1}$  in equation (3.15), the solution obtained is;

$$FV^{-1} = \begin{bmatrix} \frac{-\omega S^* \alpha_1(k_3 + \alpha_2)}{(\mu + \alpha_1)(k_2 k_3 - \kappa \alpha_2)} & \frac{\omega S^*(k_2 + \alpha_2)}{k_2 k_3 - \kappa \alpha_2} & \frac{-\omega S^*(\kappa - k_2)}{k_2 k_3 - \kappa \alpha_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(3.16)

The eigenvalues of equation (3.16) are finally computed from

 $|FV^{-1} - \lambda I| = 0$ . The results obtained are;  $\lambda_1 = \frac{-\omega S^*(\alpha_1(k_3+\alpha_2)}{(\mu+\alpha_1)(k_2k_3-\kappa\alpha_2)}, \lambda_2 = 0$ and  $\lambda_3 = 0$ . Since the Alcohol-Free equilibrium of the model is given by  $E^0 = [\frac{\Lambda}{\mu}, 0, 0, 0, 0]$ , the Jacobian matrix is evaluated at the AFE to obtain  $\lambda_1 = \frac{-\omega\Lambda\alpha_1(k_3+\alpha_2)}{\mu(\mu+\alpha_1)(k_2k_3-\kappa\alpha_2)}, \lambda_2 = 0$  and  $\lambda_3 = 0$ . The maximum modulus or dominant eigenvalue therefore defines the alcohol reproduction number,  $R_0$ . That is the spectral radius of the Jacobian matrix,  $\rho(FV^{-1})$ . From  $\rho FV^{-1}$ , the alcohol reproduction number  $R_0$  is therefore given by;

$$R_0 = max \mid b_0 \mid = max[\mid \lambda_1 \mid, \mid \lambda_2 \mid, \mid \lambda_3 \mid] = \frac{\Lambda \omega \alpha_1(b_1)}{\mu(\mu + \alpha_1)(b_2)} \quad (3.17)$$

where; $b_0 = FV^{-1} - \lambda I$ ,  $b_1 = k_3 + \alpha_2$ ,  $b_2 = k_2k_3 - \kappa\alpha_2$ ,  $\frac{\Lambda\omega\alpha_1}{\mu(\mu+\alpha_1)}$  is the average secondary infections arising from the light drinking class while  $\frac{k_3+\alpha_2}{k_2k_3-\kappa\alpha_2}$  is the average secondary infections arising from the heavy drinking class.

#### 3.3.3 Local Stability Analysis of AFE

**Theorem 3.3.1.** The AFE point  $E^0$  is locally asymptotically stable if  $R_0 < 1$  and unstable if  $R_0 > 1$ 

*Proof.* The AFE states obtained are given by;

 $S^0 = \frac{\Lambda}{\mu}$  and  $E^0 = [S^0, E^0, V_1^0, V_2^0, A^0] = [\frac{\Lambda}{\mu}, 0, 0, 0, 0]$ . In order to prove this theorem, the Jacobian matrix of the system of equation (3.1) is obtained by differentiating each equation of the system with respect to S, E,  $V_1, V_2$  and A to obtain;

$$J = \begin{bmatrix} -\mu - \omega(V_1 + V_2) & 0 & -\omega S & -\omega S & \beta \\ \omega(V_1 + V_2) & -\mu - \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & -k_2 & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 \end{bmatrix}$$
(3.18)

Equation (3.18) is solved at the AFE point,  $E^0 = \left[\frac{\Lambda}{\mu}, 0, 0, 0, 0\right]$  and the solution obtained is;

$$J_{E^{0}} = \begin{bmatrix} -\mu & 0 & -\frac{\Lambda\omega}{\mu} & -\frac{\Lambda\omega}{\mu} & \beta \\ 0 & -\mu - \alpha_{1} & \frac{\Lambda\omega}{\mu} & \frac{\Lambda\omega}{\mu} & 0 \\ 0 & \alpha_{1} & -k_{2} & \kappa & 0 \\ 0 & 0 & \alpha_{2} & -k_{3} & 0 \\ 0 & 0 & \tau_{1} & \tau_{2} & -k_{1} \end{bmatrix}$$
(3.19)

As can be seen from the Jacobian matrix in equation (3.19), the first eigenvalues are;  $-\mu$ , and  $-(\beta + \mu) = k_1$ . The following reduced matrix is used to evaluate the other eigenvalues:

$$A = \begin{bmatrix} -\mu - \alpha_1 & \frac{\Lambda\omega}{\mu} & \frac{\Lambda\omega}{\mu} \\ \alpha_1 & -k_2 & \kappa \\ 0 & \alpha_2 & -k_3 \end{bmatrix}$$
(3.20)

The characteristic polynomial of the Matrix  $J_{E^0}$  is obtained from

 $|A - \lambda I| = 0$ . Thus  $|A - \lambda I|$  is given by;

$$P(\lambda) = \lambda^{3} + a_{1}\lambda^{2} + a_{2}\lambda + a_{3} = 0$$
 (3.21)

where;  $a_1 = k_2 + k_3 + \mu + \alpha_1$ ;  $a_2 = k_2 k_3 + k_2 \mu + k_3 \mu + k_2 \alpha_1 + k_3 \alpha_1$ ;  $a_3 = (\mu + \alpha_1) k_2 k_3 + \frac{\Lambda \omega \alpha_1}{\mu} + \kappa \alpha_2$ 

To determine the necessary and sufficient condition for the existence of negative real roots, the trace and determinant of the equation (3.19) are

obtained. The trace at DFE is obtained by summing the elements on the main diagonal and the solution obtained is given by;

$$Tr(E^0) = -[2 \ \mu + \alpha_1 + k_1 + k_2 + k_3] < 0$$

and the determinant at DFE is given by;

$$k_1[-\mu(\mu + \alpha_1)(k_2k_3 - \kappa\alpha_2) + \Lambda\omega\alpha_1(k_3 + \alpha_2)] > 0$$

and on substitution with  $R_0$ ; that is from equation (3.17) let  $\Lambda \omega \alpha_1(k_3 + \alpha_2) = \mu R_0(\mu + \alpha_1)(k_2k_3 - \kappa \alpha_2)$  the following is obtained;

$$k_1\mu[-(\mu+\alpha_1)(k_2k_3-\kappa\alpha_2)+R_0(\mu+\alpha_1)(k_2k_3-\kappa\alpha_2)].$$

Let  $(\mu + \alpha_1)(k_2k_3 - \kappa\alpha_2) = A$ , then the solution obtained is;

$$k_1 \mu A[R_0 - 1] > 0$$

when  $R_0 < 1$ , thus the AFE is locally asymptotically stable.

In light of this, the analysis comes to the conclusion that the AFE is locally asymptotically stable whenever  $R_0 < 1$ . That is, given a small alcoholic population, each alcoholic in the entire time frame of alcoholism, will produce on average less than one drinker when  $R_0 < 1$ . This implies that alcohol abuse vanishes in the population when  $R_0 < 1$ . This is because media awareness might have been well implemented hence further minimization of alcohol abuse cases.

# 3.3.4 Local Stability Analysis of the Endemic Equilibrium(EEP) of the Model)

When alcoholism persists in the community, the endemic point of the model is reached. The system of equation (3.1) is solved in terms of the force of infection at the steady state  $\lambda^*$  to determine the prerequisites for the presence of an equilibrium where alcohol misuse is pervasive in the population. When the right side of equation (3.1) is set to zero and it is noted that at equilibrium,  $\lambda = \lambda^*$ , that is, the equations are solved in terms of the alcoholic force  $V_1$  or  $V_2$  and the following result is obtained:

$$S^{*} = \frac{\Lambda\beta\omega\alpha_{1}\xi(V_{1}+V_{2}) - \Lambda k_{1}(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}{\beta\omega\alpha_{1}\xi(V_{1}+V_{2})(\mu+\omega(V_{1}+V_{2}))}$$

$$E^{*} = \frac{\Lambda\beta\omega\alpha_{1}\xi(V_{1}+V_{2}) - \Lambda k_{1}(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}{\beta\alpha_{1}\xi(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))}$$

$$V^{*}_{1} = \frac{\Lambda\beta\omega\alpha_{1}\xi k_{3}(V_{1}+V_{2}) - \Lambda k_{1}k_{3}(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}{\beta\xi(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}$$

$$V^{*}_{2} = \frac{\Lambda\beta\omega\alpha_{1}\alpha_{2}\xi(V_{1}+V_{2}) - \Lambda k_{1}\alpha_{2}(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}{\beta\xi(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}$$

$$A^{*} = \frac{\Lambda\beta\omega\alpha_{1}\xi(V_{1}+V_{2}) - \Lambda k_{1}(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}{\beta k_{1}(\mu+\alpha_{1})(\mu+\omega(V_{1}+V_{2}))(k_{2}k_{3}-k\alpha_{2})}$$
(3.22)

#### **3.3.5** Endemic Equilibrium in Terms of $R_0$

The endemic equilibrium is expressed in terms of the reproduction number by substituting some of the constants that make up the reproduction number in place of those in equation (3.12). This procedure yields the following results;

$$S^{*} = \frac{\Lambda\beta\xi(V_{1}+V_{2})\mu R_{0} - \Lambda^{2}k_{1}(\mu+\omega(V_{1}+V_{2}))}{\beta(V_{1}+V_{2})\xi\mu R_{0}(\mu+\omega(V_{1}+V_{2}))}$$

$$E^{*} = \frac{\Lambda\beta\omega\mu R_{0}\xi(V_{1}+V_{2}) - \Lambda^{2}\omega k_{1}(\mu+\omega(V_{1}+V_{2}))}{\beta\mu R_{0}\xi(\mu+\omega(V_{1}+V_{2}))(\mu+\alpha_{1})}$$

$$V_{1}^{*} = \frac{\beta(V_{1}+V_{2})k_{3}\mu R_{0}\xi - \Lambda k_{1}k_{2}(\mu+\omega(V_{1}+V_{2}))}{\beta\xi(\mu+\omega(V_{1}+V_{2}))}$$

$$V_{2}^{*} = \frac{\beta\alpha_{2}(V_{1}+V_{2})\mu R_{0}\xi - \Lambda k_{1}\alpha_{2}(\mu+\omega(V_{1}+V_{2}))}{\beta\xi(\mu+\omega(V_{1}+V_{2}))}$$

$$A^{*} = \frac{\beta(V_{1}+V_{2})\xi\mu R_{0} - \Lambda k_{1}(\mu+\omega(V_{1}+V_{2}))}{\beta k_{1}(\mu+\omega(V_{1}+V_{2}))}$$
(3.23)

where;  $\varepsilon = \tau_1 k_2 + \tau_2 k_2$ 

**Theorem 3.3.2.** Endemic equilibrium exist and is locally asymptotically stable if  $R_0 > 1$ .

*Proof.* The Jacobian matrix in equation (3.17) is evaluated at the endemic steady states in equation (3.23) to obtain;

$$J^{E^*} = \begin{bmatrix} -\mu - \omega(V_1 + V_2) & 0 & -\omega S & -\omega S & \beta \\ \omega(V_1 + V_2) & -(\mu - \alpha_1) & 0 & 0 & 0 \\ 0 & \alpha_1 & -k_2 & \kappa & 0 \\ 0 & 0 & \alpha_2 & -k_3 - \lambda & 0 \\ 0 & 0 & \tau_1 & \tau_2 & -k_1 \end{bmatrix}$$
(3.24)

The eigenvalues of the system (3.24) are calculated in order to determine the necessary and sufficient prerequisites for the occurrence of the endemic equilibrium point. That is  $|J^{E^*} - \lambda I| = 0$ , which is given by;

$$\left\{\begin{array}{cccccc}
c_0 & 0 & -\omega S & -\omega S & \beta \\
c_1 & c_2 & 0 & 0 & 0 \\
0 & \alpha_1 & -k_2 - \lambda & \kappa & 0 \\
0 & 0 & \alpha_2 & -k_3 - \lambda & 0 \\
0 & 0 & \tau_1 & \tau_2 & -k_1 - \lambda
\end{array}\right\} = 0 \qquad (3.25)$$

where;  $c_0 = -\mu - \omega(V_1 + V_2) - \lambda$ ,  $c_1 = \omega(V_1 + V_2)$ ,  $c_2 = -(\mu - \alpha_1) - \lambda$ . The following characteristic equation is obtained from Jacobian in equation (3.25);

$$P(\lambda) = \lambda^{5} + A_{1}\lambda^{4} + A_{2}\lambda^{3} + A_{4}\lambda^{2} + A_{5} = 0$$
 (3.26)

where;

$$\begin{aligned} A_1 &= k_1 + k_2 + \mu + \alpha_1 + \frac{\mu(k_2k_3 + \kappa\alpha_2)(\mu + \alpha_1)(V_1 + V_2)}{\Lambda\alpha_1(k_3 + \alpha_2)}(R_0 - 1) \\ A_2 &= k_1k_3 + k_1k_2 + k_2k_3 + 2k_3 + k_1 + k_2 + \\ \frac{\mu(k_1 + k_2 + k_3 + \mu + \alpha_1)(k_2k_3 + \kappa\alpha_2)(V_1 + V_2)(\mu + \alpha_1)}{\Lambda\alpha_1(k_3 + \alpha_2)}(R_0 - 1)) \\ A_3 &= \frac{(k_1k_3 + k_1k_2 + k_2k_3)(\mu + \alpha_1)^2(k_2k_3 + \kappa\alpha_2)(V_1 + V_2)\mu}{\Lambda\alpha_1(k_3 + \alpha_2)}(R_0 - 1) + k_1k_2k_3 \\ A_4 &= \frac{\mu(k_2k_3 + \kappa\alpha_2)(V_1 + V_2)(\mu + \alpha_1)}{\Lambda\alpha_1(k_3 + \alpha_2)}(R_0 - 1)(k_1k_2k_3(1 + \mu + \alpha_1) + k_1k_2 + k_2k_3) + \mu + \alpha_1) \\ A_5 &= \frac{\mu k_1k_2k_3(\mu + \alpha_1)^2(V_1 + V_2)(k_2k_3 + \kappa\alpha_2)}{\Lambda\alpha_1(k_3 + \alpha_2)}(R_0 - 1) \end{aligned}$$

Therefore, the number of negative real roots of equation (3.24) is dependent on signs of  $A_1, A_2, A_3, A_4 \& A_5$ . This analysis can be done using the Descartes Rule of Signs of the polynomial given by equation [5].

$$P(\lambda) = A\lambda^4 + B\lambda^3 + D\lambda^2 + E\lambda + G \qquad (3.27)$$

The Descartes Rule of Signs [5] states that, the number of negative real zeros of P is either equal to the number of variations in signs of  $P(-\lambda)$ or less than this by an even number. Therefore, the maximum number of variations of signs in  $P(-\lambda)$  is four, thus the characteristic polynomial in equation (3.26) has four negative roots. Thus,

$$P(-\lambda) = -\lambda^{5} + A_{1}\lambda^{4} - A_{2}\lambda^{3} + A_{4}\lambda^{3} - A_{5} = 0$$
 (3.28)

has negative roots for  $R_0 - 1 > 0$  then the model (3.1) is locally asymptotically stable if  $R_0 > 1$ .

## 3.3.6 Summary

If  $R_0 < 1$ , then  $E_0$  is an alcohol free equilibrium of equation (3.1) and it is locally asymptotically stable. This implies that alcoholism cases disappear in the community. This may be as a result of maximum utilization of media programs in creating awareness on the effects of alcoholism. Furthermore, there exists an endemic equilibrium if  $R_0 > 1$  and is locally asymptotically stable. The existence of the endemic equilibrium indicates that, given a small alcoholic population, each alcoholic will produce on average less than one drinker when  $R_0 > 1$ . This implies that alcoholism persists in the community. That is the number of light and heavy drinkers rises further as a result of poor implementation of media programs thus the need for the government and other stakeholders such as NACADA to formulate suitable policies in targeting media awareness as an intervention measure in combating the rampant alcoholism cases in the community.

## CHAPTER 4

#### NUMERICAL SIMULATION

#### 4.1 Parameter Estimation

Utilizing MATLAB software, the system of equation (3.1) are studied. The original population estimates for S, E,  $V_1$  and  $V_2$  and A come from the 2019 reports from the Kenya Bureau of Statistics (KNBS) and the United Nations and Social Affairs [8, 20]. The initial population of the media awareness class was determined using secondary data from rehabilitation facilities in Kenya and information from the National Authority for the Campaign against Alcohol and Drug Abuse report [17]. The population of Kenya, which the United Nations estimates to be 52 million people in 2019, was used to calculate the initial conditions of the steady states |20|. The equation for this is N = S + E + V1 + V2 + A. Alcohol use is predicted to be prevalent in 31 percent of people, with alcohol addiction occurring in 13.3 percent of these people [17]. This amounts to around 2.028 million people who are alcohol dependent and 15.6 million people in the classifications  $V_1$ ,  $V_2$ , and A. Thus, the initial conditions for the variables are taken to be: S=24000000, E=14000000,  $V_1 = 12000000$ ,  $V_2 = 2000000$ , A=1600000. The parameters, their corresponding values, and the source from which they were received are all listed in Table 4.1. The parameter is directly related to the fundamental reproduction number, as indicated by the positive index.

# Table 4.1: Model Parameters and their Respective Sources

The following table summarizes the parameters of alcoholism and the respective sources obtained from.

Parameter	Value	Source
Λ	1674000	[16]
$\mu$	0.5	[9, 22]
$\beta$	0.03	Assumed
ω	0.001	Assumed
$\alpha_1$	3	Assumed
$lpha_2$	0.1	Assumed
$\kappa$	0.01	Assumed
$\sigma_1$	0.05	[15]
$\sigma_2$	0.2	[16]
$ au_1$	0.05/0.6	Assumed
$ au_2$	0.2/ $0.9$	Assumed

# 4.2 Numerical Results

The relationship between each population class in the model is shown in Figure 4.1.



#### Figure 4.1: All Alcohol Classes of the Model

Increasing  $\tau$  implies that enormous number of light and heavy drinkers who undergo media awareness and that is why the awareness class graph starts out very high before dropping somewhat and stabilizing when some people return to the susceptible class. As more people join the exposed class, the susceptible individuals become fewer over time. As some awareness individuals return to the susceptible class, the susceptible class rises somewhat before stabilizing. The exposed population decreases due to individuals joining the light drinking group. The light drinkers decreases in the first few days then stabilizes since most individuals join the awareness class. The heavy drinkers decreases further then stabilize for the remaining time. In comparison to the other classes, the awareness class has a very big enrollment. This is a result of the high rate of alcohol abusers recruited into media awareness efforts. Information can be transmitted more quickly and efficiently by being cognizant of the media. This means that most people become aware of the risks associated with alcoholism and immediately take action.

Figure 4.2 represents a case with no media awareness.



Figure 4.2: Alcoholism Model with No Media Awareness.

The the figure shows that, heavy drinking class increases rapidly due to many light drinkers advancing their drinking habits. That is individuals in the light drinking class become heavily dependent on alcohol thus becoming addicts. This further causes the number of heavy drinkers in the neighborhood to rise. Low economic output, a rise in social crime, and even an increase in mortality could follow from this. The exposed and light drinkers reduce and then stabilizes while the susceptible population reduces further as a result of transition into subsequent classes.

Figure 4.3 implies minimal media awareness rate.



Figure 4.3: Alcoholism Model with Minimal Media Awareness.

The results show that, treatment rate rises slightly, the number of people in the light drinking class increase further, and then begins to decline slightly with minimal media awareness efforts. This is because more individuals are joining the light drinking class from the exposed group and the heavy drinking class with a small number being recruited into media awareness class. The increase in the number of light drinkers implies that individuals are averagely enlightened on the dangers of alcohol abuse thus they are less dependent on alcohol and become addicts at a relatively slower rate. The heavy drinkers slightly increase and then stabilizes. This indicates that suitable media awareness programs have not been utilized in efforts to minimize the abuse of alcohol.

Figure 4.4 represents the alcohol prevalence classes plotted in the same axes.



Figure 4.4: Alcohol Prevalence of the Model.

Alcohol prevalence simply means the fraction of the population that is infected with alcoholism. The light drinkers decrease in the first few days because some individuals move to the heavy drinking class while many others move to media awareness class after which they may quit drinking since they are not highly addicted to alcohol. The media awareness class population increase in the first few days then decrease but the number in this class is less than the number in the heavy drinking class and also in the light drinking class. This situation indicates the relationship between drinking classes and treatment (media awareness). This implies that, with adequate media awareness incorporated on the drinking population, alcohol abuse stabilizes in the community. This is because, as the cases of alcoholism rises, they are immediately and effectively controlled.

## CHAPTER 5

#### CONCLUSION AND RECOMMENDATIONS

#### 5.1 Conclusion

The study aimed at formulating and analyzing a mathematical model of alcoholism incorporating media awareness as an intervention strategy. The model was formulated based on a system of differential equations. The study investigated and examined the invariant region of the model and proved that the model is biologically well-posed. After considering the initial conditions in terms of the positivity of the model, it was determined that all alcoholism state variables are positive for all time t > 0. By use of the next generation matrix, the alcohol reproduction number  $R_0$  was calculated, and this aided in determining the threshold value for the occurrence of alcoholism. By use of Jacobian matrix method, local stability analysis of the model was performed at two distinct equilibrium points namely; Alcohol Free equilibrium and Alcohol/endemic equilibrium point.

Using the Jacobian matrix method, the conditions for the existence of the AFE point were investigated and it was shown to be locally asymptotically stable whenever  $R_0 < 1$ . This suggests that alcoholism becomes extinct in the community. Using the Descartes method the necessary and sufficient condition for the existence of the AEP were investigated and it was shown that the endemic equilibrium of the model exists and is positive if  $R_0 > 1$ , hence the prerequisites for its existence were satisfied. This demonstrates how alcoholism spreads through a community, highlighting the importance of making the most of media awareness resources.

Numerical analysis of the model indicated that increase in media awareness programs reduces alcohol prevalence in the community. The awareness degree is varied as in Figure 4.1, 4.2 and 4.3 and with high rate of media awareness, alcohol prevalence decrease. With minimal media awareness, there is an increase in the number of light drinkers. With no awareness then more individuals remain in heavy drinking class thus increased alcohol dependency and addiction. It is clear from the study that the exposed class and media awareness have a correlation. The interaction between the exposed class, light and heavy drinkers facilitates the understanding of the spread, nature and occurrence of alcoholism. Additionally, increase in media awareness programs plays a key role in further reduction of the light and heavy drinkers. Thus, the study concluded that the exposed class has a greater impact in minimizing the numbers of light and heavy drinkers in the presence of media awareness. Further, the study concluded that media awareness is the best intervention strategy in curbing alcoholism in a community but it cannot alter its spread.

#### 5.2 Recommendations

The study recommends that the Kenya government should encourage media awareness programs on alcohol abuse by strengthening mass media campaigns against alcohol consumption. The government through narcotic bodies such as NACADA should enforce laws that discourage media campaigns that advocate for the sale of alcohol to the most vulnerable groups. Additionally media houses should offer guidance and counseling services via radio and television to enlighten its citizen on alcoholism. More importantly the Kenyan government should endeavor setting up more rehabilitation centers where addicts can be sensitized on dangers of alcohol abuse and rehabilitated either through detoxification process or other forms of treatment.

# 5.3 Recommendation for future works

- The study suggests additional research be done on the effects of media awareness campaigns on other substances that are often abused in Kenya, with a focus on the influence of the exposed class.
- The study advocates that the future works should target other effective treatment and prevention strategies to combat drug abuse.
- Future researchers should work together with narcotic bodies such as NACADA, so as suitable policies against alcoholism can be formulated and implemented.

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